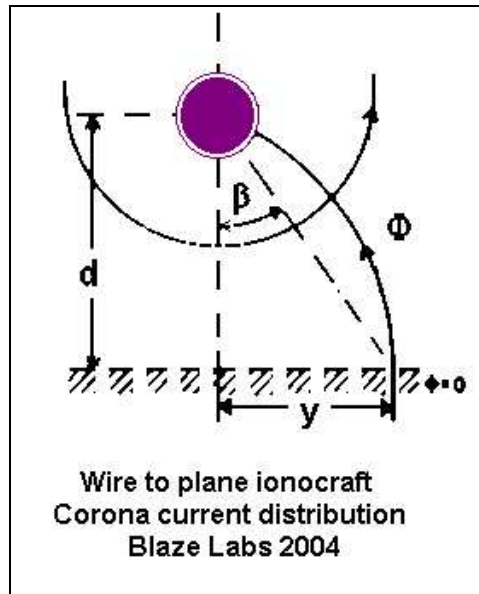


Full analysis & design solutions for EHD Thrusters at saturated corona current conditions

Category: Ionocrafts & Lifters
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This paper deals with analysis & design solutions for single stage multiple corona EHD Thrusters. The charge flow is simplified as multiple paths of unipolar ions drifting all together in the form of an ion cloud with mobility μ and negligible diffusion effects. A voltage source of voltage V is applied between the corona wire and collector grid. The corona is a discharge where ionisation is non thermal.

In the above diagram, the top conductor is the corona wire (not to scale), d is the vertical airgap distance from wire to plane grid, and y is the effective lateral distance over which the cloud spreads out during its journey to the collector. β is called the distribution angle and is a measure of spreading out of the ion cloud taking into account its interaction with neutral air molecules and ion space charge.

It can be assumed that, all charge transport through the gap is carried by charged particles having the same polarity as the corona as described in detail in <http://blazelabs.com/l-intro.asp>. The ion flow lines coincide with the electric field lines, but, the electric field distribution is strongly dependent on the ion space charge. At high currents or corona saturation currents, the current distribution $j(\beta)$ is of a modified Laplacian form which was earlier found by Warburg in 1899, that it closely follows the so called Warburg \cos^m distribution, given by:

$$j(B) = j_0 \cos^m(\beta) \dots\dots m = 4.82 \text{ for positive corona and } m = 4.65 \text{ for Negative Ionocrafts}$$

If one plots the Warburg distribution it can be clearly seen that for angle range of 60° to 65° , the current density falls sharply from 4% towards 1%, indicating that the field lines further out than point y , at which $\beta \geq 65^\circ$ are not acting upon the ion cloud. Due to the small difference in m for different polarities, the angle β for positive ion clouds at which this threshold occurs is slightly less, calculated to be just one degree less, that is 64° .

This first rule, clearly indicates that implementing collector grids, which laterally exceed $2d \cdot \tan(\beta)$ will not have any beneficial effect upon the resulting thrust, and result only in additional 'dead' weight. Knowing β , we can now conveniently express y , half the width of the ion cloud in terms of the height d of the wire above the plane, since

$$K \tan(\beta) = y/d \quad (1)$$

$$K \tan(65^\circ) = y/d$$

$$y = 2.1 Kd \dots\dots \text{half spread width for negative emitter corona wire} \quad (2)$$

for positive ions

$$K \tan(64^\circ) = y/d$$

$$y = 2 Kd \dots\dots \text{half spread width for positive emitter corona wire} \quad (3)$$

K is the Borg-Sigmond coefficient which depends on the actual geometry of the electrodes. See appendix.

Effective ion cloud cross sectional area

Thus, the effective area of the negative or positive ion clouds at the grid per unit length ℓ of the grid becomes:

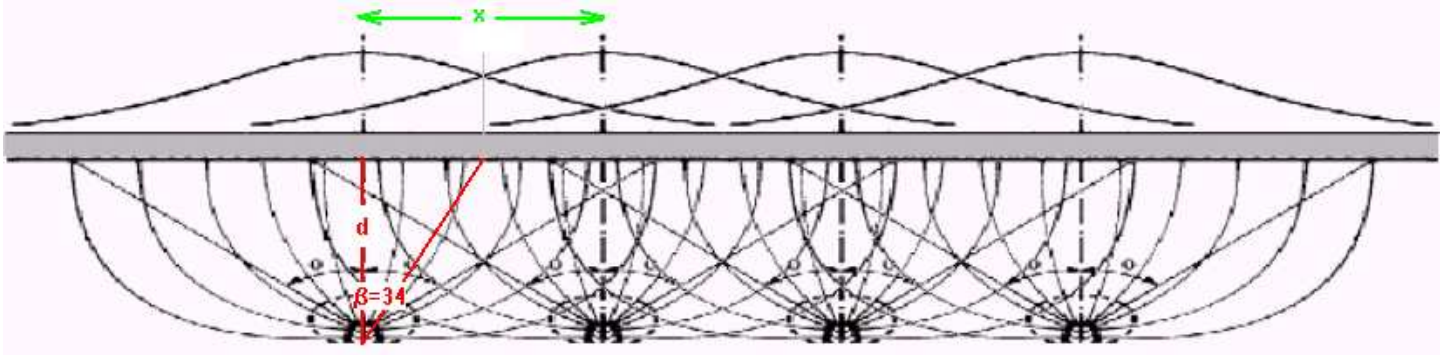
$$A_n = 2 y \ell = 4.2 K d \ell \quad \dots \text{for negative corona} \quad (4)$$

$$A_p = 2 y \ell = 4 K d \ell \quad \dots \text{for positive corona} \quad (5)$$

So, effective area is independent of actual grid lateral width (given that the width $2y \geq 4.2Kd$ or $4Kd$ respectively)

Xavier's Law for multi-corona element spacing

There is a minimum distance 'x' between corona sources (point or wire), at which the effective force of the 2 sources will be less than twice of a single source. In fact for very close sources, the effective force will be the same as for one source and will virtually behave as one effective point or wire. This is due to the fact that the current density at any point in the cloud cannot exceed the saturated current limit j_0 . Ideally, with multiple parallel sources, assuming no ions are lost to the surroundings, the collector should see a constant current density of j_0 all over its surface. In the below diagram, the current density at the collector from each corona wire is superimposed. The total current density at each position on the collector would be the sum of all the plots. As you can see, the region of least current density will always be at midpoint between the emitters, at which the collector should ideally receive $\frac{1}{2} j_0$ from each source. Now solving for $\cos^{4.82}(\beta) = 0.5$, we get $\beta = 30^\circ$. So the optimal distance between 2 elements is $x = 2y = 2Kd \tan(30) = 1.15 d$. This applies to all twin point to plane geometries, and also to twin wire to plane.



For 4 parallel elements, this becomes a little bit more complex, because the 3rd & 4th elements at the extreme ends will add another pair of components at the middle of the collector at an angle $(\tan^{-1} [3y] / \tan^{-1} [1y]) * \beta = 1.59\beta$

$$j_{max} = j_0(2\cos^{4.82}(\beta) + 2\cos^{4.82}(1.59\beta)) \leq j_0 \quad \text{which gives } \beta \sim 34^\circ, \text{ and a spacing of } x = 2Kd \tan(34^\circ) = 1.35d \dots\dots\dots (6)$$

For six or more parallel elements, the β factors will result in angles greater than 65° and thus have virtually no effect, so $x=1.35d$ can be taken as the maximum distance even for higher numbers of parallel elements. If the wires are set closer than this, the corona current density at the wire diminishes so that at no point does the current density ever exceeds the saturated value j_0 for a single wire. In the limit where $x=0$, the saturation current will be equally divided in two, and they will therefore act as a single wire.



Lifter designed with parallel corona wires spaced at $1.3d$

Derivations for maximum pressure, force, air flow, air velocity & corona saturation current

The ion flow is driven through the air gap by the electric field and braked by the collision and friction with the neutral gas molecules. Ion acceleration in the wire to plane geometry is negligible, and thus all the electric energy from the field is ultimately transferred to the neutral molecules. We can thus integrate the force qE along all field lines, and take the effective force projected over the plane grid. This has been worked out by Sigmond et al, and the total force exerted by the corona current i over a gap distance d is given by:

$$F = i d / \mu \dots\dots F \text{ is force in Newtons, } I \text{ current in Amps, } d \text{ air gap in metres, } \mu = \text{ion mobility in air}$$

This shows that the vertical component of the force is independent of the actual ion path and electric field. Again we have a small difference between negative and positive ions, due to a small variation in mobility for different polarities, $\mu_n = 2.7 \text{ E-4 m}^2/\text{Vs}$, $\mu_p = 2.0 \text{ E-4 m}^2/\text{Vs}$, thus we have:

Ion cloud velocity $v_{ion} = \mu E \dots$ **resulting in ion cloud velocities > 100m/s (7)**

$$F = i d / 2.7 \text{ E-4} \dots \text{ for negative corona} \quad (8)$$

$$F = i d / 2.0 \text{ E-4} \dots \text{ for positive corona} \quad (9)$$

This force, is equal to the momentum transfer rate between the fast ion cloud to the almost stationery neutral air molecules, and can be used to calculate gas flow and velocity at the lower side of the grid. If we assume a free flowing air stream through the collector grid effective area A , with average air flow S and velocity v (m/s), we have:

$$S = v/A$$

$$F = i d / \mu = \rho S v = \rho S^2 / A, \dots\dots\dots \rho \text{ is the air density in kg/m}^3, S = \text{flow rate in litres/sec}$$

$$S = (i A d / \mu \rho)^{1/2} \quad (10a)$$

$$S \text{ (CFM)} = 2.1186 * (i A d / \mu \rho)^{1/2} \quad (10b)$$

$$v = S/A = (i d / \mu \rho A)^{1/2} \quad (11)$$

Now, the total maximum pressure acting over the active grid area A (having dimensions: length ℓ , width $2y$), is equal to :

$$P = F/A$$

$$P = i d / (A \mu)$$

$P = i d / (2y \ell \mu) \dots$ writing y in terms of d , $y = 2.1Kd$ or $y = 2Kd$ for the respective corona polarities we have:

$$P_{max} = i d / (4.2 K d \ell \mu) = i / (4.2 K \mu \ell) = j / (4.2 K \mu) \dots \text{ for negative coronas} \quad (12)$$

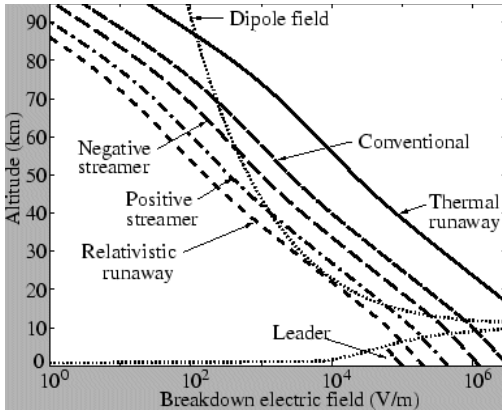
$$P_{max} = i d / (4 K d \ell \mu) = i / (4 K \mu \ell) = j / (4 K \mu) \dots \text{ for positive coronas} \quad (13)$$

This shows that pressure generated does not depend on the length ℓ , but only on saturation current density j and the Borg-Sigmond electrode geometry coefficient K .

Now the pressure gradient is equal to coulombs force acting on each ion.

From Gauss law, we have $\delta p / \delta z = \epsilon_0 * E * (\delta E / \delta z) \dots$ where E varies from zero to the streamer breakdown voltage

$$\text{Integrating for total pressure increase } \Delta P_{max} = 1/2 \epsilon_0 E_{max}^2 \text{ or } 1/2 \epsilon_0 (V/d)^2 \quad (14)$$



The breakdown in the ionised air gap occurs by means of a totally different mechanism than the conventional electron avalanche, namely by corona streamers. Streamers are ionisation waves which can propagate as narrow channels through regions where the electric field is less than the conventional breakdown voltage for air (30kV/cm). In the plot below, you can see the conventional Townsend avalanche breakdown voltage at sea level is approximately 3E6 V/m, but once we have corona streamers in action, this drops to either 1.1E6 V/m for negative streamers, or even worse, about 0.6E6 V/m for positive streamers. This self-propagation as we know, is due to highly nonuniform electric fields which result from significant gradient in current density, or space charge.

Note that again, we have another variation between streamer breakdown voltage (V/d) for negative and positive coronas, where $E_n \sim -11\text{kV/cm}$ or -1.1E6 V/m and $E_p \sim +6\text{kV/cm}$ or $+0.6\text{E6 V/m}$. However this average breakdown voltage varies with pressure, temperature, humidity, altitude and lateral wind speed. So, it is recommended to use actual streamer breakdown V/d values from experimental data.

We can now equate the above pressure equations:

$$P_{\max} = j_{\max} / (4.2 K \mu) = \frac{1}{2} * \epsilon_0 E_n^2 \quad \text{for negative coronas} \quad (15)$$

$$P_{\max} = j_{\max} / (4 K \mu) = \frac{1}{2} * \epsilon_0 E_p^2 \quad \text{for positive coronas} \quad (16)$$

Thus the saturation corona current per unit length (per metre) j_{\max} in each case is given by:

$$j_{\max} = 2.1 * K \mu \epsilon_0 E_n^2 \quad \text{for negative coronas } E_n = -V/d \quad (17)$$

$$j_{\max} = 2 * K \mu \epsilon_0 E_p^2 \quad \text{for positive coronas } E_p = +V/d \quad (18)$$

This shows that an upper limit for current density exists which is independent of actual ionocraft size, given it's driven under saturated corona current conditions. The same applies for pressure generated.

$$\text{Total current consumption } i_{\max} = j_{\max} * \ell \quad (19)$$

$$\text{Total power consumption } P = i_{\max} * V \quad (20a)$$

$$\text{It is also given by } P = F_{\max} * v_{\text{ion}} \quad (20b)$$

Substituting for j_{\max} in $F = id/\mu = j \ell d / \mu$, we get

$$F_{\max} = 2.1 K \epsilon_0 V^2 * (\ell / d) \quad \text{for negative coronas} \quad (21)$$

$$F_{\max} = 2 K \epsilon_0 V^2 * (\ell / d) \quad \text{for positive coronas} \quad (22)$$

$$\text{Thrust in gF } T = (1000 / 9.8) * F_{\max} \quad (23)$$

Using equations (23) & (20a), the thrust to power ratio in g/Watt assuming $g=9.8\text{m/s}^2$:

$$T/P = (1000 / 9.8) * F_{\max} / (V * i_{\max}) = 102 / (\mu E) \quad \text{where } E = E_p \text{ or } E_n \text{ depending upon polarity} \quad (24)$$

Machine efficiency = Mechanical energy output / Electrical energy input, assuming thruster reaches air velocity

$$\text{Conversion efficiency \%} = 100 * F_{\max} * v / (i_{\max} * V) \quad (25)$$

$$\text{Performance } F_{\max} \text{ per unit length} = 2.1 K \epsilon_0 V^2 / d \quad \text{for negative coronas} \quad (26)$$

$$\text{Performance } F_{\max} \text{ per unit length} = 2 K \epsilon_0 V^2 / d \quad \text{for positive coronas} \quad (27)$$

$$\text{Fan performance (CFM/Watt)} = S(\text{CFM}) / P \dots (28)$$

Practical worked estimates for an Ionocraft working at +40kV (positive corona), as an array of 10 elements each 20cm.

From $E_p \sim +0.6\text{MV/m}$, we know that for maximum thrust, the gap distance d must be $40\text{kV}/(6\text{kV/cm}) = 6.7\text{cm}$ or 0.067m

Total element length = 10 elements * 0.2m = 2m

Eqn(3)... Ion spread width $y = 2 * K * d = 2 * 1 * d = 13.3\text{cm}$ or 0.133m

Eqn(5) Ion cloud area reaching grid = $A_p = 2 * y * \ell = 2 * 0.133 * 2 = 0.53\text{m}^2$

Eqn(6) ... Distance between parallel corona elements $x = 1.3d = 1.3 * 0.067 = 0.087\text{m}$ or 8.7cm

Eqn(7) ... Ion cloud velocity $v_{ion} = \mu E = 2\text{E-}4 * 0.6\text{E}6 = 120\text{m/s}$

Eqn(18)... Saturated corona current per metre $j_{max} = 2\mu\epsilon_0 E_p^2 = 2 * 2\text{E-}4 * 8.8542\text{E-}12 * 0.6\text{E}6^2 = 1.28\text{mA/m}$ or $12.8\mu\text{A/cm}$

Eqn(19) ... Current consumption for full length $i = j * \ell = 1.28 * 2 = 2.55\text{mA}$

Eqn(10a) air flow rate $S = (i A d / \mu\rho)^{1/2} = (2.55\text{E-}3 * 0.53 * 0.067 / (2\text{E-}4 * 1.2))^{1/2} = 0.61\text{ litres/sec}$

Eqn(10b) air flow rate CFM = $0.61 * 2.1186 = 1.29\text{CFM}$

Eqn(11) air velocity $v = S/A = (id / \mu\rho A)^{1/2} = (2.55\text{E-}3 * 0.067 / (2\text{E-}4 * 1.2 * 0.53))^{1/2} = 1.15\text{m/s}$

Eqn(22)..... $F_{max} = 2\epsilon_0 V^2 * (\ell / d) = 2 * 8.8542\text{E-}12 * 40\text{E}3^2 * (2 / 0.067) = 0.85\text{N/m} = 87.18\text{gF}$

Eqn(16) $P_{max} = j / (4\mu) = 1.28\text{E-}3 / (4 * 2\text{E-}4) = 1.6\text{Pa}$ (or $P_{max} = 1/2 \epsilon_0 E_{max}^2 = 1.6\text{Pa}$) (or $P_{max} = F_{max} / A_p = 1.6\text{Pa}$)

Eqn(20a/b) Total power consumption = $i * V = 2.55\text{mA} * 40\text{kV} = 102\text{W}$ (or $F_{max} * v_{ion} = 0.85 * 120 = 102\text{W}$)

Eqn(24)..... $T/P = 102 / (\mu E_p) = 102 / (2\text{E-}4 * 0.6\text{E}6) = 0.85\text{g/Watt}$

Eqn(25)..... Conversion efficiency = $100 * F_{max} * v / (i V) = 100 * 0.85 * 1.15 / 102 = 0.96\%$ (or $100 * v / v_{ion} = 100 * 1.15 / 120 = 0.96\%$)

Eqn(28) Fan Performance = $S (\text{CFM}) / P = 1.29 / 102 = 0.013\text{CFM/Watt}$.

So, this ionocraft will produce a 87.18gF thrust, and run at 2.55mA , 102Watts .

A similar sized lifter will produce 43.6gF thrust, and run at 1.28mA , 51Watts .

This ionocraft can also be used as an EHD hovercraft or EHD speaker generating a total pressure of 1.59Pa .

As a fan, it is rated at 1.29CFM performing at 0.013CFM/Watt .

As an EHD air pump it will move about 2212.69 litres of air per hour

Quick reference:

$\mu_n = 2.7\text{E-}4\text{m}^2/\text{Vs}$

$\mu_p = 2.0\text{E-}4\text{m}^2/\text{Vs}$

$E_n \sim -11\text{kV/cm}$ or $-1.1\text{E}6\text{V/m}$

$E_p \sim +6\text{kV/cm}$ or $+0.6\text{E}6\text{V/m}$

Warburg current distribution $j(B) = j(0) \cos^m(\beta)$

$m = 4.82$ for positive corona

$m = 4.65$ for negative corona

Experimental distribution angle $\beta = 65^\circ$ for negative corona ionocrafts

Experimental distribution angle $\beta = 64^\circ$ for positive corona ionocrafts

$K = \text{Borg-Sigmond generalised geometric coefficient}$

($K=9/8$ for planar geometry, $K=1$ for multiwire ionocrafts, $K=0.5$ for lifters)

$\epsilon_0 = 8.8542\text{E-}12\text{F/m}$

air density $\rho = 1.2\text{kg/m}^3$

$g = 9.8\text{m/s}^2$

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